

KHAIRA COLLEGE, KHAIRA, BALASORE

DEPARTMENT OF PHYSICS

QUESTION BANK

UG 3RD Sem - CC - V

Answer all questions

1- Answer the following : [1mark]

- a) The period of sine function is _____ .
- b) The product of an odd function and even function is _____ .
- c) $\beta(9,15) - \beta(15,9) =$ _____ .
- d) For a stationary wave, the points where there is no displacement of particles are called _____ .
- e) At _____ point a function is not analytic.
- f) $\operatorname{erf}(x) + \operatorname{erf}(-x) =$ _____ .
- g) $\gamma(n+1) =$ _____ .
- h) Can a non periodic function be expanded in Fourier series?
- i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \dots =$ _____ .
- j) If $f(x)$ is odd in $(-\pi, \pi)$ the Fourier co-efficient, $a_n =$ _____ .
- k) $y^n + \frac{y}{x^3} = 0, x = 0$ is _____ singular point. (regular/irregular)
- l) $\beta(m, n) = \beta(n, m)$ (True / False).
- m) $P_n(1) =$ _____ .
- n) What is the value of $H_2(x)$?
- o) Write generating function of Hermite's Polynomial.
- p) The solution of Laplace equation is $V(x) =$ _____ .

2- Answer the following (Very short type) :- [1.5 marks]

- a) Find the period of $\cos \frac{\pi x}{2L}$.
- b) Define even and odd function with examples.
- c) Define ordinary point.
- d) Find the value of Legendre Polynomial $P_n(x)$ at $x = 0$.

- e) Write Rodrigue's formula for Hermitie's polynomials. Find its value for $n = 0$.
- f) Find the singular point of Laguerre differential equation.
- g) Find the value of $r(\tau)$.
- h) Find the order and degree of the differential equation
- $$\left(\frac{d^2y}{dx^2}\right) + 2\left(\frac{dy}{dx}\right)^2 + 5y = 0$$
- i) What is the solution of the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y}$, of $u(0,y) = e^{-2y}$.
- j) Show that β -function is symmetric.
- k) Write Fourier Cosine series.
- l) Evaluate $\beta\left(\frac{3}{2}, \frac{1}{2}\right)$.
- m) Define irregular singular point.
- n) Write Dirchelet condition.
- o) Prove that $P_n(1) = 1$.
- p) Prove that $P_0(x)$ and $P_1(x)$ are orthogonal t each other.
- q) Define standard error.
- r) State two properties of Hermite Polynomial.
- s) Write Laplace equation in Cartesian coordinate system.
- t) Define normal modes of vibration in string.

3- Answer the following (Sort type) :-

[2 marks]

- a) For a period function $f(x)$, write the expression for Fourier series and find the 1st co-efficient 'a₀'.
- b) Find the singular point of the differential equation
- $$x^2y'' + 2xy' + \lambda y = 0.$$
- c) Find the Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$.
- d) Prove that $P_n(-x) = (-1)^{n+1} P_n(x)$.
- e) Explain regular singular point for a differential equation
- $$y'' + P(x)y' + Q(x)y = 0$$
- f) Evaluate $\int_0^\infty x^3 e^{-x^2} dx$.

- g) Define error function. Write two characteristics.
- h) What are harmonics and overtones?
- i) Find the value of $\gamma\left(\frac{3}{4}\right) \times \gamma\left(\frac{1}{4}\right)$.
- j) Prove that $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}} dx = 0$.
- k) Find the period of the periodic function

$$F(t) = r \sin\left(\frac{t}{T} + \frac{\pi}{4}\right)$$
.
- l) State Parseval's identity.
- m) Find cosine series for $f(x) = x$ in $0 \leq x \leq \pi$.
- n) State orthogonality condition of Legendre's Polynomials.
- o) Show that $H_{2s}^1(0) = 0$.
- p) Prove that $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$.
- q) Define Gamma function.
- r) Express the integral $\int_0^{\infty} \frac{x^3}{(x+1)^5} dx$ in terms β function.
- s) Define spherical harmonic, find them in terms of Legendre Polynomials.
- t) Write Rodrigue formula for Hermite Polynomial.

4- Answer the followings (Long type) :-

[6marks]

- a) Find the Fourier series for $f(x) = x \sin x$ in the interval $(-\pi, \pi)$ and show that .

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \text{ ---}$$
- b) State and prove Parseval identity.
- c) Find series solution of Hermite's differential equation.
- d) Derive Rodrigue's formula for Lagendre polynomials.
- e) Derive expression for Associated Legendre differential equation.
- f) Define γ -function and β -function. Derive a relationship between them.

- g) Write Laplace's equation in spherical polar co-ordinates and find a solution for it by using method of separation of variables.
- h) Write the expression for wave equation. Find the solution for it.
- i) Find Fourier series of $f(x) = x^2$ in $[0, 2\pi]$.
- j) Find the complex form of Fourier series
 $f(t) = \sin t, 0 < t < T$.
- k) Show that the Hermite Polynomial are generated by generating function, $g(x, t) = e^{2xt - t^2}$.
- l) Prove that Orthogonal Property of Hermite Polynomial.
- m) Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$
- n) i) Find $\sqrt{\left(\frac{1}{4}\right)}, \sqrt{\left(\frac{3}{4}\right)}$
ii) Show that $\beta(m, n) = \frac{m+n}{n} \beta(m, n+1)$.
- o) Write and solve Laplace's equation in spherical coordinate.
- p) Applying Laplace's equation discuss conducting sphere in an external uniform electric field.

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